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SELF-MODELING PARAMETER DISTRIBUTION BEHIND A DETONATION WAVE

V. N. Okhitin UDC 534.222.2

An analytic solution exists only in the planar case to the self-modeling problem of the parameter distribution behind a stationary detonation-wave front in a perfect gas [1]. Numerical solutions have been derived for waves of spherical and cylindrical symmetries by the use of various equations of state for detonation products DP [2-4]. There are also analytic approximations for the distributions behind the fronts of symmetrical detonation waves DW in condensed explosives [4, 5], which are interesting for use as initial conditions in solving more complex detonation problems. Numerical calculations show that the behavior of the DP from gaseous explosives [6] and condensed ones of various densities [4, 5] can be described quite accurately from the isentropic relation between the pressure p and density ρ for a perfect gas with various values of the adiabatic parameter γ. It is therefore of interest to derive analytical relationships for the self-modeling parameter distribution behind a stationary DW front for various types of symmetry and for the equation of state for a perfect gas whose adiabatic parameter varies over a wide range. Tight specifications are laid down for the analytic relationships on account of their use in numerical calculations. In particular, they should satisfy the asymptotic solutions in the region of the DW front and at the boundary with the central region at rest, and they should also describe the numerical solutions with sufficiently high accuracy.

Here we derive analytic relations for the self-modeling parameter distribution behind a stationary DW front in the Chapman–Jouguet mode for various forms of symmetry satisfying these requirements.

A system of ordinary differential equations in the self-modeling variable $\zeta = r/t$ describes the parameter distribution behind a stationary DW front [7]:

$$\frac{du}{d\zeta}\left[\frac{(\zeta-u)^2}{c^2}-1\right]=\frac{\nu u}{\zeta},\quad (\zeta-u)\frac{du}{d\zeta}=\frac{c^2}{\rho}\frac{d\rho}{d\zeta},$$

where ρ, u, and c are the density, mass velocity, and speed of sound, r and t are independent variables for the distance from the point of detonation initiation and time, and ν is the symmetry parameter, which takes the values 0, 1, and 2 correspondingly for planar, cylindrical, and spherical DW.

We introduce the dimensionless linear coordinate r/R, where R is the coordinate of the DW front, and use the fact that $c \sim \rho^{(\gamma-1)/2}$ for a perfect gas, whereupon the system can be reduced to

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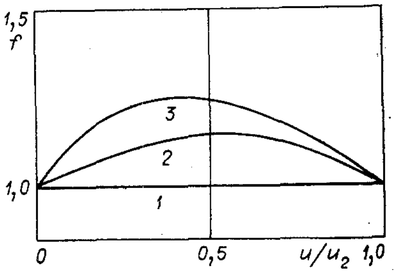

Fig. 1

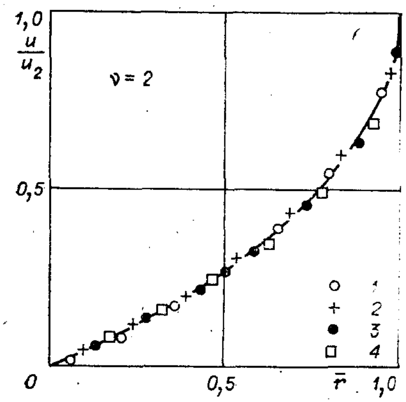

Fig. 2

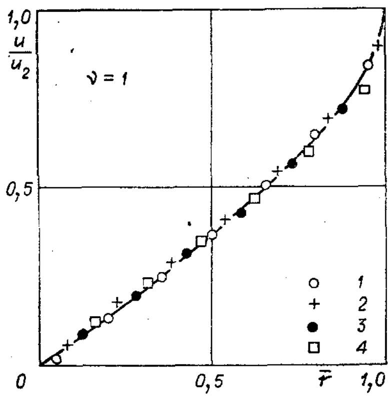

Fig. 3

$$\frac{du}{d(r/R)}\left[\frac{(Dr/R-u)^2}{c^2}-1\right]=\frac{\nu u}{r/R},\quad \frac{Dr/R-u}{c}\frac{du}{d(r/R)}=\frac{2}{\gamma-1}\frac{dc}{d(r/R)}, \tag{1}$$

where D = R/t is the DW front speed.

In the planar case, $\nu = 0$, and it follows from the first equation in (1) that $(Dr/R - u)/c = 1$, so $c = Dr/R - u$.

From the second equation in (1) we get a known analytic solution for a planar DW:

$$u = u_2 - 2(c_2 - c)/(\gamma - 1),$$

where the subscript 2 relates to the parameter values at the DW front.

Also, u = 0 at the boundary with the central region of stationary parameters, and the speed of sound there is given by

$$c_3 = c_2 - (\gamma - 1)u_2/2.$$

The boundary of the region of rest is a line of weak discontinuity, so on the basis of the detonation parameters [7]

$$c_2 = D\frac{\gamma}{\gamma+1}\left[1+\frac{p_1}{\rho_1 D^2}\right],\quad u_2 = D\frac{1}{\gamma+1}\left[1-\gamma\frac{p_1}{\rho_1 D^2}\right]$$

we get as follows for the coordinate of the stationary zone in a planar DW:

$$\frac{r_3}{R} = \frac{c_3}{D} = \frac{1}{2}\left(1+\gamma\frac{p_1}{\rho_1 D^2}\right). \tag{2}$$

Here p is pressure and the subscripts 1 and 3 relate to the initial parameters of the medium ahead of the DW front and at the boundary of the region at rest.

The second term in the parentheses in (2) is usually neglected because of its smallness, but it may be appreciable in a gas detonation.

In a planar DW, the combination $f = (Dr/R - u)/c$ in (1) is equal to one and is independent of the properties of the DP, i.e., of the adiabatic parameter. One assumes that in the case of a symmetrical DW, this function will also not be dependent on γ. Numerical solutions have been derived for DW of various symmetries with adiabatic parameters varying from 1.1 to 3, and these show that f(u) is almost independent of γ. The form of the relationships is shown for various types of symmetry in Fig. 1 (ν = 0, 1, and 2 for lines 1-3 correspondingly).

In that case, it is not difficult to integrate the second equation in (1) with limits from the DW front to the boundary of the rest region, with $\int f(u)du$ calculated graphically, and thus to obtain an expression for the speed of sound in the stationary-parameter region $c_3 = c_2 - (\gamma - 1)\bar{f}u_2/2$.

The mean values $\bar{f}$ in the cylindrical and spherical cases are 1.105 and 1.174 correspondingly, and for any type of symmetry the value is given closely by

$$\bar{f} = (6\nu+15)/(4\nu + 15).$$

Then we have as follows for the stationary-region parameters:

$$\frac{r_3}{R}=\frac{c_3}{D}=\frac{1}{D}\left[c_2-\frac{\gamma-1}{2}\frac{6\nu+15}{4\nu+15}u_2\right]$$

or on the basis of the expressions for c_2 and u_2

$$\frac{r_3}{R}=\frac{c_3}{D}=\left(\frac{r_3}{R}\right)_{\nu=0}\left[1-\frac{\gamma-1}{\gamma+1}\frac{2\nu}{4\nu+15}\frac{1-\gamma p_1/\rho_1 D^2}{1+\gamma p_1/\rho_1 D^2}\right],$$

where $(r_3/R)_{\nu=0}$ is the radius of the stationary zone in the planar case, which is defined by (2). As $p_1/\rho_1 D^2$ is small, the latter expression can be rewritten as

$$\frac{r_3}{R}=\frac{c_3}{D}=\left(\frac{r_3}{R}\right)_{\nu=0}\left[1-\frac{\gamma-1}{\gamma+1}\frac{2\nu}{4\nu+15}\left(1-2\gamma\frac{p_1}{\rho_1 D^2}\right)\right]. \tag{3}$$

Expression (3) approximates the numerical results with an accuracy of about 0.5% throughout the range in γ.

If f(u) is independent of γ, it follows from the first equation in (1) that the mass-velocity distribution in the DW is also independent of the properties of the DP and can be represented as

$$u/u_2=\varphi(\bar{r}),$$

where $\bar{r} = (r - r_3)/(R - r_3)$ is the dimensionless relative radius in the Taylor decompression wave behind the DW front, which varies from 0 to 1.

Figures 2 and 3 show numerical results for the spherical and cylindrical cases correspondingly in the form of mass-velocity distributions for DP for γ = 1.1, 1.5, 2, 3 (points 1-4 correspondingly). It is clear that the results lie virtually on the same curves for all these γ.

System (1) shows [2] that $du/dr \to \infty$ in the region of the DW front in the spherical and cylindrical cases, while $du/dr = 0$ as we approach the central stationary region, while all higher derivatives tend to infinity. All these conditions are satisfied by the function

$$\varphi(\bar{r})=u/u_2=1-(1-\bar{r}^{\alpha})^{\beta}\quad\text{for}\quad 1<\alpha<2\ \text{and}\ 0<\beta<1.$$

The numerical results enable one to select values for α and β, which are

$$\alpha=1.05,\quad \beta=2/3\quad\text{for}\quad \nu=1,$$
$$\alpha=1.1,\quad \beta=1/2\quad\text{for}\quad \nu=2.$$

The mass-velocity distributions constructed with these α and β are shown by solid lines in Figs. 2, 3.

With $\alpha = \beta = 1$, $(\bar{r})$ leads to the known analytic solution for a planar DW. This enables us to suggest the following relations for α and β for an arbitrary symmetry type:

$$\alpha=(20+\nu)/20,\quad \beta=2/(\nu+2).$$

Then we have the following general relationship for the velocity distribution in a stationary DW:

$$u=u_2\left\{1-\left[1-\left(\frac{r-r_3}{R-r_3}\right)^{(20+\nu)/20}\right]^{2/(\nu+2)}\right\}\quad\text{for}\quad r_3\leqslant r\leqslant R. \tag{4}$$

Figure 1 shows that f(u) varies only slightly in DP [the maximum change in f(u) occurs in the spherical case and is about 20%]. Therefore, it can be considered as constant to a first approximation and equal to the mean value $\bar{f}$. Then we integrate the second equation in (1) with limits from r_3 to the current radius r and from r_3 to R to get

$$\bar{f}u=2(c-c_3)/(\gamma-1),\quad \bar{f}u_2=2(c_2-c_3)/(\gamma-1).$$

This means that

$$u/u_2=(c-c_3)/(c_2-c_3).$$

We then have the following approximate relation for the speed of sound in the DP on the basis of (4):

$$c = c_3 + (c_2 - c_3)\left\{1 - \left[1 - \left(\frac{r - r_3}{R - r_3}\right)^{(20+\nu)/20}\right]^{2/(\nu+2)}\right\} \text{ for } r_3 \leqslant r \leqslant R. \quad (5)$$

The pressure and density of the DP are determined from the speed of sound via the isentrope. The error in describing the numerical calculations is about 1% for the most sensitive parameter: the pressure.

Therefore, analytic relationships (2)-(5) describe the numerical solution with high accuracy for the distribution of the parameters behind a stationary detonation-wave front in a perfect gas and satisfy the asymptotes of the exact solution.

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DESCRIPTION OF SHOCK-WAVE PROCESSES IN A TWO-PHASE MEDIUM CONTAINING AN INCOMPRESSIBLE PHASE

V. A. Vakhnenko and B. I. Palamarchuk

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The motion of a two-phase medium is analogous to that of a perfect gas with a certain effective adiabatic parameter within the framework of the one-velocity model when the volume proportion of condensed phase is small [1-3]. If on the other hand no constraint is placed on the volume proportion, the basic hydrodynamic equations contain it as a variable additional to those in the analogous gasdynamic equations. This substantially complicates solving the nonstationary hydrodynamic equations and has led to the need to develop the methods given in [4, 5].

Here we propose a method of transforming the variables that leads to complete analogy between the equations for a perfect gas and those for a two-phase medium with any volume occupied by the condensed phase. It is shown that the motion of a two-phase medium in the transformed coordinate system is completely analogous to that of a perfect gas, which means that the methods developed for perfect gases can be used to solve shock-wave problems.

The scope for the method is demonstrated by reference to the strong explosion state in a two-phase medium.

1. Basic Concepts. Consider a homogeneous two-phase medium consisting of condensed and gas phases uniformly distributed in the volume. We assume as follows: 1) The condensed phase is incompressible, 2) the gas obeys the equation of state for a perfect gas with constant values for the specific heats, 3) the partial pressure of the condensed phase is negligibly small, 4) the speeds of the condensed phase and gas are equal, and 5) there is no reaction between the components.

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